Magic of Topology

Impossible Science Topic - Can a shape have only one side?

For students in grades K through 6th

'Impossible Science' lessons aim to be 75-90 minutes long for grades 3 through 6, and 60-75 minutes long for grades K through 2. The final 15 minutes for K-2 will be filled with a 'wind down' drawing activity (see "Evaluate" section for details).

NOTE: Delete italicized text when finished with Lesson Plan development.

Objectives

By the end of this lesson, students will:

- **know** the field of topology, also known as "rubber band geometry", is the study of shapes and how the can change without being broken
- **understand** how changes to a shape's structure and orientation can alter how they are perceived by viewers
- **be able to** design and craft a mathematical illusion using their knowledge of topology, perspective, orientation, and available supplies

Big Ideas:

- Shapes can be **deformed** and **transformed** without permanently changing their properties. The field in which mathematicians explore this fact is called **topology.**
- By altering a shape's **orientation**, we can change how we perceive the shape and its size.
- We can also change our perception of the shape by changing our **perspective.**
- We can take advantage of topology, orientation, and perspective to create mathematical illusions

Essential Questions:

- How does the appearance of a shape affect our efforts to describe the shape?

- In what ways can we change how a shape appears without changing the shape?
- How does the orientation of a shape affect the way we see it?
- How does our perspective relative to a shape affect the way we see it?
- When we see something that seems to be visually impossible, what does our brain do?

Engage:

Warm Up Activity [3 minutes]

Ask the students to generate a list of shapes. As they come up with new shapes, ask them to explain the key features of the shape which help them identify it as that shape.

- *Conversation may get interesting when students start to mention 4-sided shapes, like rectangles, squares, rhombuses, trapezoids, etc. Let them mull over the challenge of distinguishing between these figures before providing assistance!*

Think | Pair | Share [5 minutes]

- Think [1-2 min]: Give students the following question prompts and ask them to consider them individually
    - What is the smallest number of sides a shape can have? Why?
    - *Grades 2 to 6 - have students write their answers down in their workbook.*
    - *Grades K to 2 - have students draw as many shapes as they can which have the fewest number of sides they can think of.*
- Pair [1-2 min]: Students pair up and take turns sharing their thoughts on the questions. They ask each other questions after sharing and create their combined responses to the question prompts.
- Share [2-3 min]: Bring the larger group together and let the pairs take turns summarizing their combined responses.
    - *Grades 2 to 6 - have students write and draw their final answers in their workbooks*

Stronger students will likely name the triangle, or get creative with curved surfaces.

Impossible Science Demo - String Handcuffs [5 minutes]

"Today, I am going to make a daring escape from rope handcuffs, but first I need the help from a volunteer"
Once you have volunteer, complete the rope handcuffs trick with string (steps below)

- Tie one person's hands together with a string by securing a loop around each hand.
- *"We are not just going to tie our own hands together, but we are going to connect our string handcuffs to each other, thus making it impossible for us to separate from each other."*

"Teach like a Magician" Suggested Script tagged with [TLAM]:

Step 1:

- *[TLAM]: As you can see, we are completely stuck together"*

Step 2:

- *[TLAM]: "If either of us pulls or slides our string, we are unable to find a way out of the other person's rope. To all appearances, we are secured together and unable to separate from each other."*
    - Demonstrate to students that you seem to be inseparable.

Step 3-4: (do not repeat these steps to students)

- With as much sleight of hand as you and your partner are comfortable with, free yourselves from each other by completing the handcuff trick (steps with pictures below)

.***Step 5:***

- *[TLAM]: "Tah dah! I have magically untangled the string and freed us from an eternity of forced friendship!"*
- Lead discussion with students to try and explain what they think just happened.
    - *Invite students to model their predictions in their workbooks with sketches and words.*

Lead into next exercise, stressing that we are going to learn more about topology to help us understand how this trick was possible

Explore:

Activity 1 - Mobius Strips [20 minutes]

Students will work individually on this task. Distribute the materials for "Activity 2" on the Materials page of the lesson plan.

For instructions on these exercises, watch through the video here, in advance of teaching the lesson.

- Before completing the mobius strip, have students connect the ends of a piece of paper without a twist, making a normal cylinder, then have them cut it down the middle
    - They should get two separate loops, nothing magical about that!
- Now have them twist the narrow strip of paper 360 degrees, tape the ends together and cut it in half, and see what happens!
    - Two separate loops again
- Next, Students will mimic the steps followed in the Impossible Science Video Linked above.
    - Twist the strip of paper long-ways by 180 degrees.
    - Tape the two short ends of the Mobius Strip together.
    - Using a marker, students will trace a line along the Mobius Strip.

- Ask the group to discuss what they think would happen if the students were to cut the Mobius Strip along the line they just traced.
    - *Most students will say they anticipate creating 2 separate Mobius Strips*
    - *If possible, write down answers on the board or on chart paper and keep a tally of who expects each outcome.*
- Have students cut along their traced lines, producing a larger Mobius Strip!
- Challenge faster students to do it again, and see if the results repeat themselves.

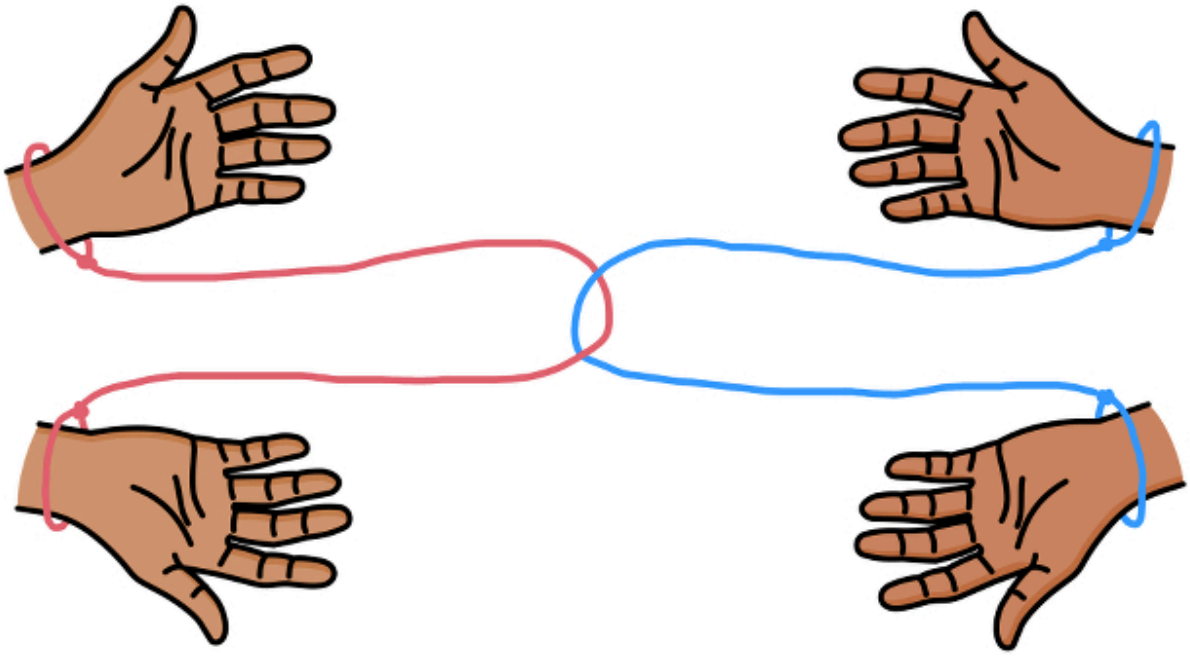
Discuss the results of this activity. Highlight the fact that the Mobius Strip is an interesting example of topology, as it seems to have two sides even though it only has one. And, we were able to alter it without changing its behavior.

- Have students twist a strip of paper long-ways 180 degrees by twisting one end **towards** their bodies. Then tape the short ends together.
- Twist another strip of paper long-ways 180 degrees by twisting one end **away from** their bodies. Then tape the short ends together.
- Tape the two Mobius Strips together so that one's side is perpendicular to the other's, forming a sort of "+" sign where they're taped together.
    - *This activity works best when the two strips are secured carefully with tape! Help those who lack the fine motor skills refine their taping before moving on.*
- Using a marker, trace a line around both of the Mobius Strips
- As the group to discuss what they think would happen if the students were to cut both strips in half along the lines they just traced.
    - *If possible, write down answers on the board or on chart paper and keep a tally of who expects each outcome.*
- Have students cut along their traced lines, producing two, interlocked hearts.
- Challenge faster students to cut the strips in half again and see if the results repeat themselves.

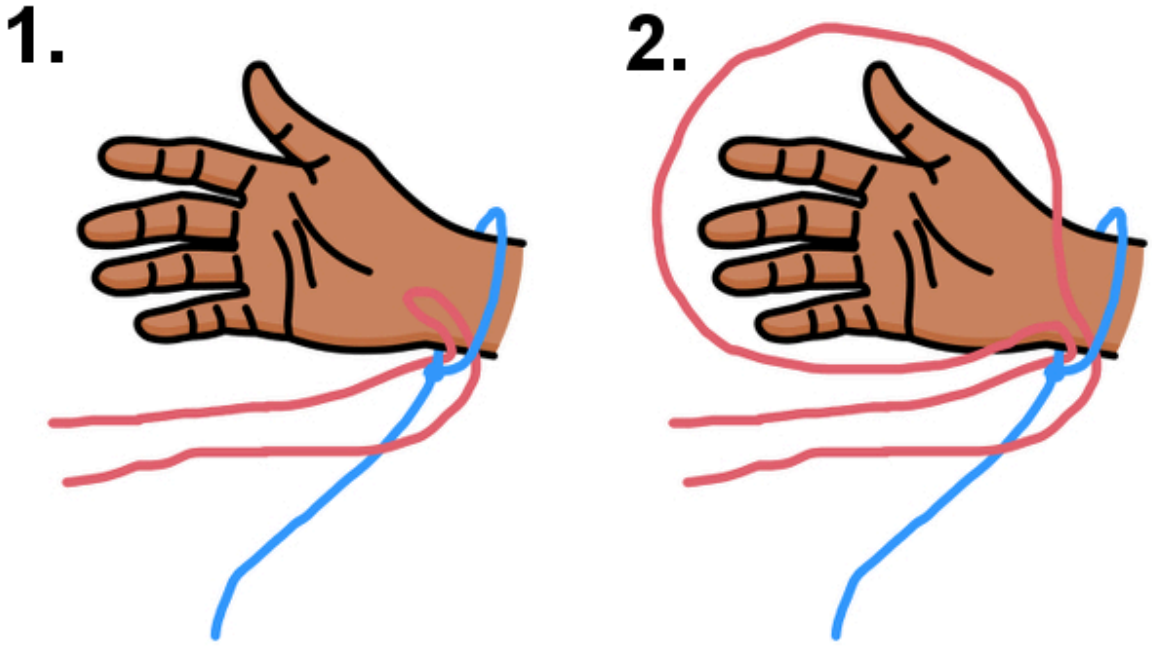
Note: Younger learners may lack the fine motor skills to successfully tape and cut their Mobius Strips. Be attentive to this, and provide support where needed.

Activity 2 - Geometric Illusions and String Handcuff Puzzle [5 minutes]
Students will work in pairs to try and solve the handcuff trick you have done as a magic trick. Distribute the "Activity 1" Materials.

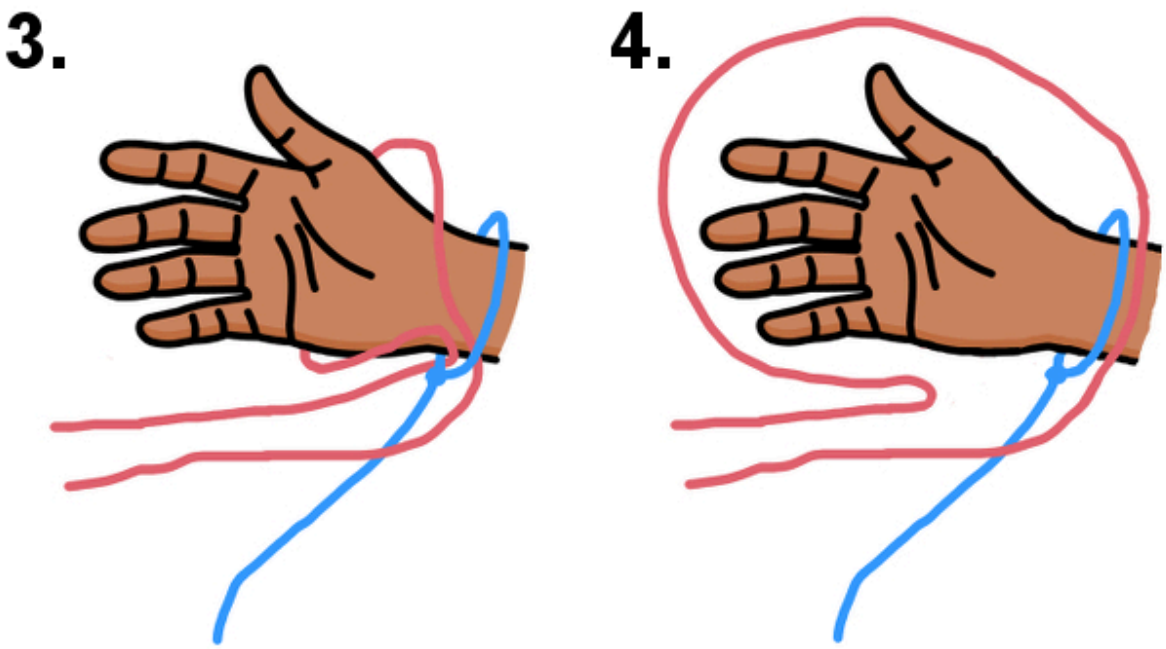
- Cut two lengths of string, each approximately 4' long
- Tie one person's hands together with a string by securing a loop around each hand.
- Loop the second string around the first person's string. Then, tie the second person's hands together. Once done, the two 'string handcuffs' should be connected by the loops between each person's hands.



- After a few minutes, remind students that this is an issue of orientation and perspective. To break free, you need to reorient the geometry to see how to get loose.
- One handcuffed person slides her loop of string through one of the other handcuffed person's wrist loops, pulling through a good amount of string in the process.



- Then, take the loop and pull it over the other person's hand. At this point, the loop is no longer looped around the other string. Simply pull it free of the other person's wrist loop to free both people.



Activity 3 - Exploring Orientation and Perspective [15 minutes]

Students will work in groups of 3-4 on this activity. Distribute the materials for "Activity 3" on the Materials page of the lesson plan.

- Using each object individually, have students place the objects in such a way that they can change their perspective by:
    - Moving closer/farther to the object
    - Walking around the object
- As they change their perspectives, ask students to share with their teammates what the object looks like. They should note changes such as:
    - Perceived size relative to other objects
    - 2D shapes (i.e., it looked like a rectangle from over here, but now it looks like a square over here)
- Challenge the students to place the objects in size order (smallest to largest) in such a way that, when viewed from a certain perspective, they appear to be in the exact opposite order (largest to smallest).
    - *This is an example of a forced perspective illusion. Students will need to line the objects up on a flat surface that seems to taper off into the distance (table,*

hallway) and then look at them head-on to create the illusion that the smaller objects, which are closer to them, are larger.

Explain:

Lecture with Manipulatives [10 minutes]

Using the provided materials and experiments, explain the “Big Ideas” of the lesson. Be sure to highlight the following:

- Today, we learned about topology, also known as "rubber band geometry." This is the study of shapes and how they can change without being broken. Shapes can be deformed and transformed in surprising ways, as seen with the Mobius Strip and the String Handcuffs.
- Changes in orientation and perspective can alter how we perceive shapes. When we looked at objects from different angles, they appeared to change size and shape. Our forced perspective illusion with the objects on the table showed us how we can trick our eyes into seeing things differently.
- Our eyes can play tricks on us because of how our brains work to process and interpret what we’re seeing. We can use this, along with our understanding of topology, orientation, and perspective, to create illusions based on math and science.

Elaborate:

Experiment - Mathematical Illusions [20 minutes] (3rd-8th)

Students will work individually on this experiment. Distribute the materials for "Experiment" on the Materials page of the lesson plan.

- Review each of the included mathematical illusions with students. Highlight how the illusions use intersecting lines and black/white color intersections, to create the illusions.
    - Hering Illusion - Are the lines bending?
    - Ponzo Illusion - Which line is longer?
    - Ebbinghaus Illusion - Which orange circle is bigger?
    - Kanizsa Triangle - Is there a white triangle drawn?
    - Cafe Wall Illusion - Are the lines crooked?
    - Hermann Grid Illusion - Do you see the gray dots?
    - Checkerboard Blister - is there a bump in the paper?
- In their workbooks, have students roughly sketch on a design for a mathematical illusion.
- Distribute grid paper and crafting materials
- Challenge students to create their own mathematical illusions. Guide their work with probing questions about topology, orientation, perspective, and so on.
    - *Some students may want to work together on this, and that is encouraged!*

Note: *For younger learners, this activity will be quite challenging. As a method of scaffolding the activity, have them complete the following task instead.*

- On your piece of grid paper, trace your hand.
- Using your pencil or crayons, draw straight lines along the grid lines of your piece of paper. But, when you get to the outline of your hand, curve the lines.
- Repeat this process for every parallel line on your piece of paper, curving the lines over the outline of your hand.
- When completed, your hand outline should magically appear to be three dimensional!
- Color in your creation!

Share Out [remaining time]:
Have students take turns sharing their illusions with the class. Encourage them to explain how their illusion works.

Modify / Extend:

Extension Activity 1 - Forced Perspective Photography
Students work in groups of 4 for this task.

- Invite students to use what they've learned about forced perspective to create a funny picture.

Evaluate:

In their workbooks, have students sketch their final mathematical illusion design. Encourage older students to explain how their illusions work.

Materials

Impossible Science Demo (1 per class)

- String
- Scissors

Activity 1 (1 per student pair)

- String (approximately 8' total)
- Scissors

Activity 2 (1 per student)

- 2 long strips of paper
- Tape
- Scissors

Activity 3 (1 per group of 3-4 students)

- Small manipulatives from Impossible Science kit of various sizes (4 total)
    - Pipe cleaners
    - Cups
    - Tape container
    - Crafting materials

Experiment (1 per student)

- Printouts of example Illusions (1 set per class)
- Gridded paper
- Crafting materials (paper, tape, glue, scissors)
- Pencils, markers, colored pencils, crayons, etc.

Modify 1

- n/a

Extension 1

- Digital camera